Enhanced Time-Difference-of-Arrival Tracking Scheme Using Dynamic Reference

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There is an increasing interest in investigating one promising new technology – ultra-wide band (UWB) – to design the tracking system for both space and commercial applications. The applications of this research include:

- In space, to track lunar/Mars rovers in early exploration phases. After a Moon/Mars base has been established, it is necessary to track the rover or astronaut's positions while working around the base.
- On Earth, to track firefighters in a forest or building.

To exploit the fine time resolution of the UWB signal, a tracking scheme Time-Difference-of-Arrival (TDOA) is employed for the tracking system design. The conventional TDOA algorithm requires one receiver centered at the origin of the coordinate system as a static reference. The study shows that the tracking resolution can be improved by using the dynamic reference.

Description of the Problem

The TDOA tracking algorithm currently used in the tracking system design is developed in "An efficient closed-form localization solution from time difference of arrival measurements Acoustics, Speech, and Signal Processing" (Chan, YT, Ho, KC, 1994). Assume that one transmitter is located at an unknown location (x_0, y_0) in two-dimensional space and $M+1$ receivers are located at positions $\{(0,0),(x_1,y_1),(x_2,y_2),\ldots,(x_M,y_M)\}$, which are assumed to be known precisely. Further, assume that measurements of the relative time delays $\{d_1,d_2,\ldots,d_M\}$ between the arrival of the transmitted signal at receiver $(0,0)$ (the reference receiver) and each of the other locations $(x_1,y_1),\ldots,(x_M,y_M)$ are available. If the propagation velocity of the signals is given by the constant c, it can be shown that the following system of linear equations is satisfied:

$$\mathbf{G}_0\mathbf{u}_0 = \mathbf{h}_0 , \qquad (1)$$

where

$$\mathbf{u}_0 = \begin{bmatrix} x_0 \\ y_0 \\ r_0 \end{bmatrix}, \quad r_0 = \sqrt{x_0^2 + y_0^2}, \quad \mathbf{G}_0 = -2\cdot\begin{bmatrix} x_1 & y_1 & cd_1 \\ x_2 & y_2 & cd_2 \\ \vdots & \vdots & \vdots \\ x_M & y_M & cd_M \end{bmatrix}, \quad \mathbf{h}_0 = \begin{bmatrix} c^2d_1^2 - x_1^2 - y_1^2 \\ c^2d_2^2 - x_2^2 - y_2^2 \\ \vdots \\ c^2d_M^2 - x_M^2 - y_M^2 \end{bmatrix}.$$

By going through three stages of the algorithm, the final estimation of the transmitter position can be obtained. The idea of this algorithm is that a nonlinear problem is transformed to a linear problem by adding a dummy variable r_0. This requires a receiver placed at $(0,0)$ as the static reference. This static reference causes larger errors in certain tracking areas.

Solution to the Problem

The solution to the problem caused by the static reference is to use the dynamic reference. Instead of using one particular receiver as the reference, the reference receiver is dynamically chosen by minimizing a cost function. We derive and present the mean-squared-error (MSE) as the cost function for a simple long-range example. In particular, we let $M = 3$, $\mathbf{Q} = \sigma^2\mathbf{I}$, and $r_0 >> \max_{1\le i\le M}\left(\max\{|x_i|,|y_i|,cd_i\}\right)$. If we consider an orbit tracking in two-dimensional space (see figure 1, r_0 is the radius of the orbit and r is the radius of the area in which the receivers are placed),

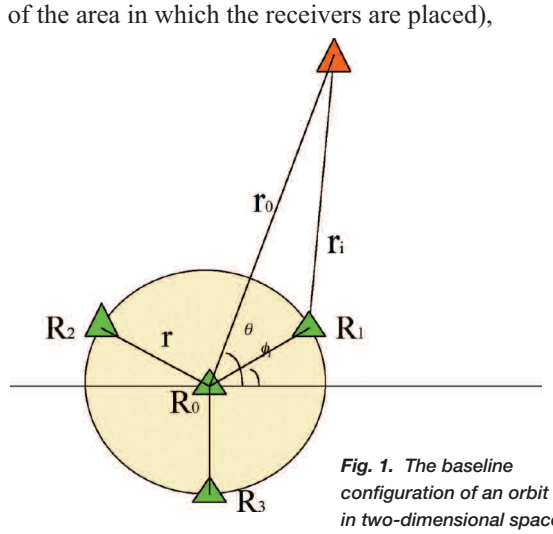

Fig. 1. *The baseline configuration of an orbit tracking in two-dimensional space.*

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and let $x_0 = r_0 \cos\theta, y_0 = r_0 \sin\theta, x_i = r\cos\phi_i$, $y_i = r\sin\phi_i$, and $r_i = \sqrt{r_0^2 + r^2 - 2r_0 r\cos(\theta - \phi_i)}$, the total MSE is given by

$$\mathrm{MSE} = \frac{c^2\sigma^2 r_0^2 \sum_{i=1}^{3}\left(a_i^2 + b_i^2\right)}{r^2\left(\sum_{i=1}^{3} a_i^2 \sum_{i=1}^{3} b_i^2 - \left(\sum_{i=1}^{3} a_i b_i\right)^2\right)}, \qquad (2)$$

where $a_i = \cos\phi_i + \frac{r_i - r_0}{r}\cos\theta$, and $b_i = \sin\phi_i + \frac{r_i - r_0}{r}\sin\theta$

It is shown that MSE is a function of parameters σ, r_0, r, θ, and ϕ_i, and it is linear to the variance σ^2 of the TDOA estimates.

The dynamic reference scheme is to choose the receiver that gives the minimum MSE as the reference when the transmitter moves into different areas.

Performance Evaluation

To evaluate the performance of the dynamic reference scheme vs. the static reference scheme, a set of two-dimensional orbit (radius 100 m) tracking simulations is conducted. The radius of the receiving baseline is 5 meters. Three receivers are evenly placed on the circle with one at the center. The transmitter moves along the orbit from 0 to 360 deg. A tracking estimate is taken for each position at every degree, so the total number of tracking samples is 360 deg.

To evaluate the performance improvement achieved by using the dynamic reference scheme, an analysis of the tracking error for TDOA noise level $\sigma = 10ps$ is shown in figure 2. It is noticed that there are two tracking error peaks from 27 to 89 deg and from 210 to 267 deg. Those two peaks are eliminated by using different receiver (pink) as the reference in the dynamic reference scheme. Overall, the simulation shows that the tracking resolution is improved from 1.7660 to 0.8757m at the TDOA noise level $\sigma = 10ps$ by using the dynamic reference.

For lower TDOA noise level $\sigma = 1ps$, a similar tracking error pattern is observed. The simulation shows that the tracking resolution is improved from 0.1728 to 0.0819m at the TDOA noise level $\sigma = 1ps$ by using the dynamic reference. The simulation results also show that the tracking error is linear to the standard deviation of TDOA data, which coincides with the analytical result in "UWB Tracking System Design for Free-Flyers" (Ni, J, et al., 2004).

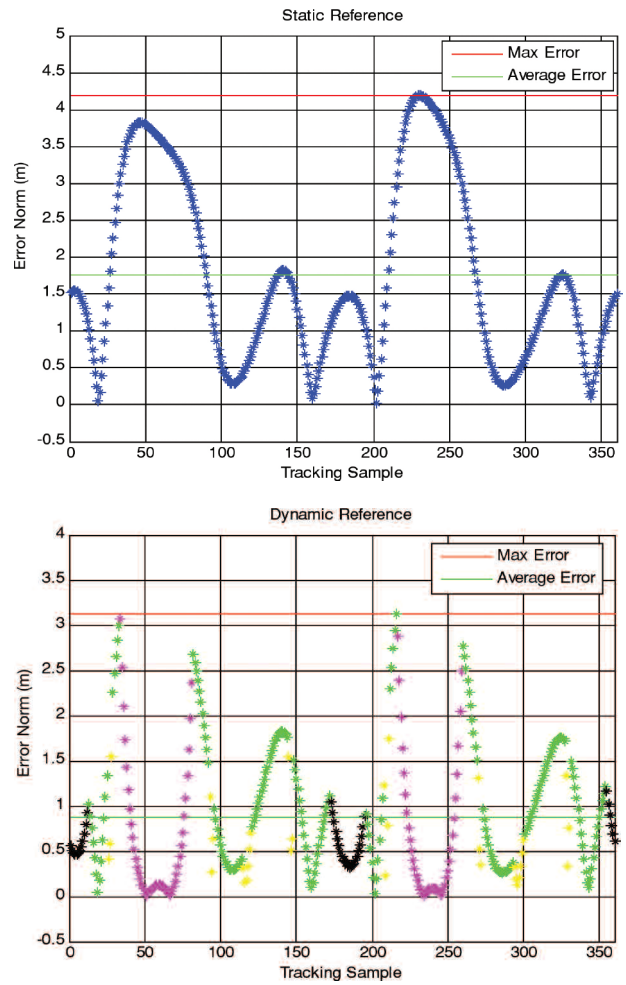

Fig. 2. Error analysis (static reference vs. dynamic reference, $\sigma = 10ps$).

Conclusion

A tracking algorithm TDOA is investigated and the dynamic reference scheme is proposed to replace the static reference scheme. Simulations show that the dynamic reference scheme can improve the tracking performance and achieve the fine resolution. The TDOA algorithm with the dynamic reference can enhance the UWB tracking system for various space applications.